#1 B GB

Since the diagonals bisect each other, LMNK is a parallelogram. That eliminates choice D, since a trapezoid is not a parallelogram. We also know the diagonals are congruent which means LMNK is a rectangle.

#2 D

If MEQT is a parallelogram, then its opposite angles are congruent. Therefore:

$$6x + 12 = 5x + y$$

Simplify → $-5x$ $\quad -5x$

$$x + 12 = y$$

If you substitute the x, y pairs from the choices into $x+12=y$, you find that choices B and D are possible solutions. To eliminate another choice, use the fact that MEQT is <u>not</u> a rectangle and therefore does not have $90°$ angles.

If $x=13$ (choice B) then:

$$\begin{aligned} m\angle E &= 6x+12 \\ &= 6(13)+12 \\ &= 78+12 \\ &= 90° \end{aligned}$$

If $x=15$ (choice D) then:

$$\begin{aligned} m\angle E &= 6x+12 \\ &= 6(15)+12 \\ &= 90+12 \\ &= 102° \end{aligned}$$

Correct choice, since not a right angle.

#3 YES, SRPQ is a rhombus

There is not enough information to determine if all four sides of SRPQ are congruent, so instead we will check to see if the diagonals are perpendicular. If they are, then SRPQ is a rhombus. Consider the $\triangle$ with side lengths 10, 24, and 26. See if the lengths form a Pythagorean triple.

$$10^2 + 24^2 = 26^2$$
$$100 + 576 = 676$$
$$676 = 676$$

Since the Pythagorean relationship holds true, $\overline{SP} \perp \overline{QR}$ and therefore SRPQ is a rhombus.

#4 $\overline{VT} = 28$

Since VWTR is a rectangle, its diagonals bisect each other and are congruent. So if $WZ = 14$, then $ZR = 14$ as well, which makes $WR = 28$. If the diagonals are equal then $VT \cong WR$ and is 28 as well.

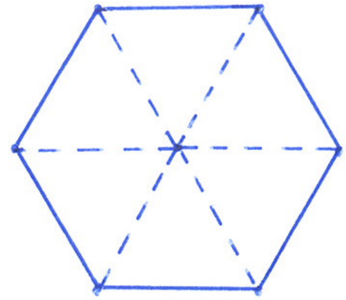
#5 62°

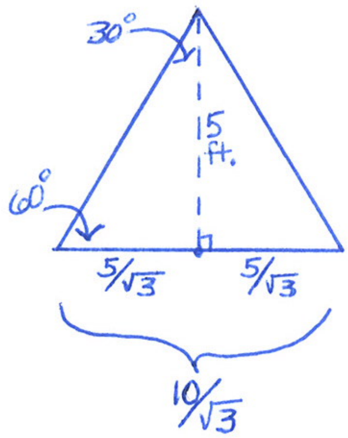
The exterior angles of a polygon always add up to 360°. Therefore:

$$x + 88 + 88 + 32 + 90 = 360^\circ$$
$$x + 298 = 360$$
$$x = 62$$

#6 86.6 sq. feet

Notice that a regular hexagon is composed of six congruent triangles. If you find the area of one Δ, you could multiply it by 6 to get the total area.





They are equilateral triangles (a regular hexagon's interior angles are 120°, so the triangles' base angles must be half that $\rightarrow$ 60°). Using the 30°-60°-90° Δ relationship (see diagram) we know that half the base is $5/\sqrt{3}$ and therefore the entire base length is $10/\sqrt{3}$.

$$\text{Area} = \frac{\text{base} \cdot \text{height}}{2} = \frac{(10/\sqrt{3})(5)}{2} = 14.434$$

$$(14.434)(6) = 86.6 \text{ ft}^2$$

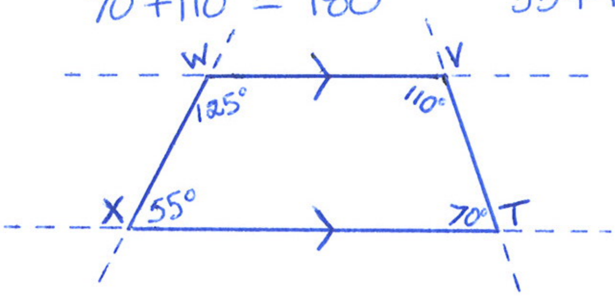
#7 55° $\longrightarrow$ trapezoid

The interior angles of any quadrilateral total 360°.

$$m\angle X + 70 + 110 + 125 = 360$$
$$m\angle X + 305 = 360$$
$$m\angle X = 55°$$

Note that both pairs of adjacent angles are supplementary.

$70 + 110 = 180$ $\qquad$ $55 + 125 = 180$

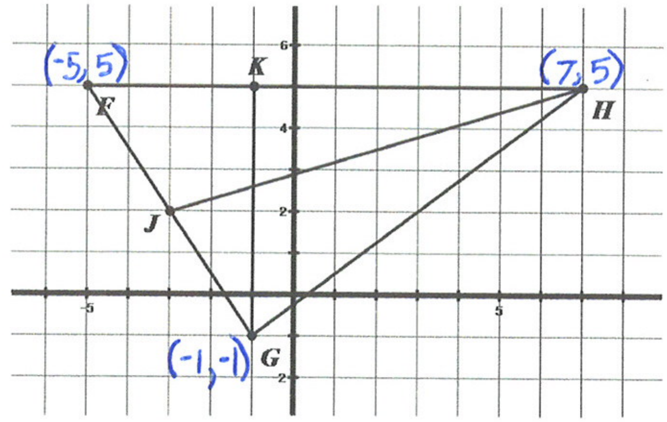


Since same-side interior angles are supplementary, the lines are parallel, which makes TVWX a trapezoid.

GB

#8

If $\overline{HJ}$ is a median, then J is the midpoint of $\overline{FG}$.



$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$J = \left(\frac{-5+-1}{2}, \frac{5+-1}{2}\right)$$

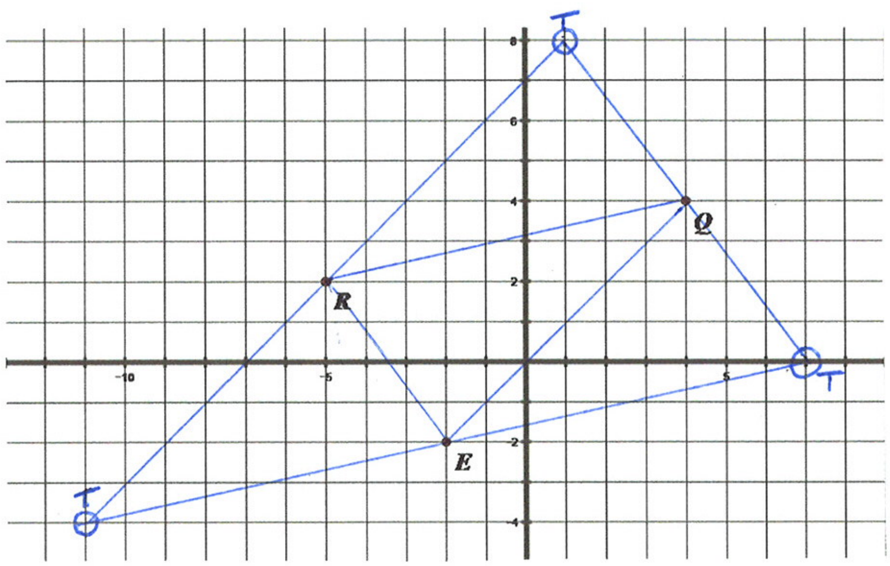
$$J = (-3,2)$$

If $\overline{GK}$ is an altitude, then it is perpendicular to $\overline{FH}$. Simply start at G on the graph and travel directly up until you intersect $\overline{FH}$ at point K. The coordinates are:

$$K = (-1,5)$$

#9

There are three possible locations for point T that will ensure the two sets of parallel sides a parallelogram requires. To find them, use the fact that parallel lines have the same slope.

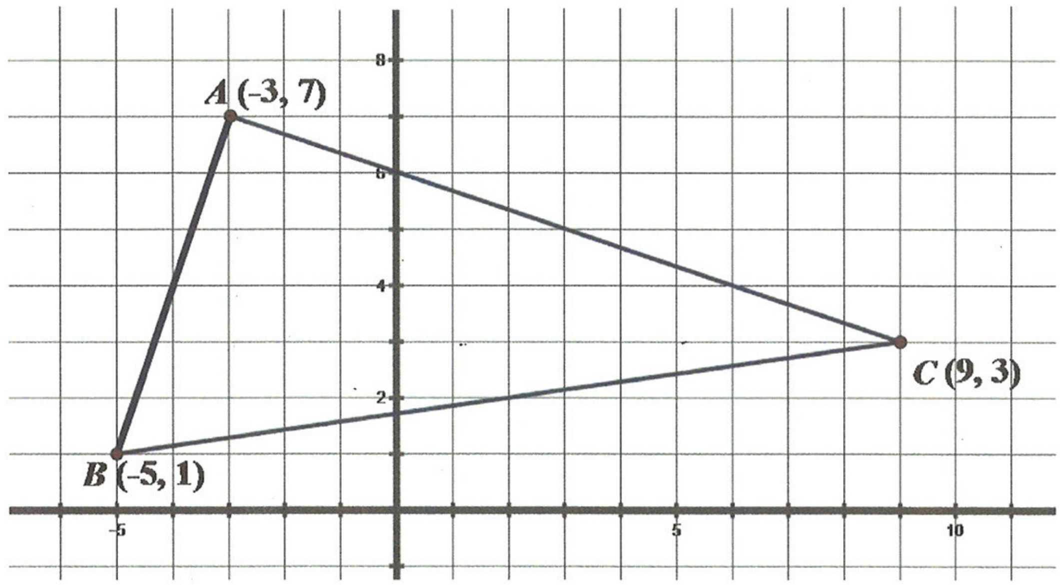


$\angle T$ opposite $\angle E$	$\angle T$ opposite $\angle R$	$\angle T$ opposite $\angle Q$
To get to Q from E, you travel 6 units up and 6 units right. For a parallel segment, travel 6 units up and 6 right from point R.	To get from R to Q, travel up 2 units and 9 units right. Follow that path from E to get another set of coordinates for T.	To get to R from Q, you travel 2 units down and 9 units left. Follow that path from E to get the last possible location for T.
$T = (1,8)$	$T = (7,0)$	$T = (-11,-4)$

#10 (2,2) GB

The circumcenter of a triangle is equidistant from its vertices, so it is the center of the circle that circumscribes it. Therefore, you must find the intersection of the triangle sides' perpendicular bisectors.

Suppose we find the ⊥ bisectors of segments AC and BC, and then find their point of intersection.



The midpoint of AC is:

$\left(\frac{-3+9}{2}, \frac{7+3}{2}\right) \longrightarrow (3,5)$

The slope of AC is:

$\frac{3-7}{9-{}^{-}3} = \frac{-4}{12} = \frac{-1}{3}$

The perpendicular bisector must contain the point (3,5) and have slope 3.

$y = mx + b$

$5 = 3(3) + b$

$5 = 9 + b$

$-4 = b$

$y = 3x - 4$

The midpoint of BC is:

$\left(\frac{-5+9}{2}, \frac{1+3}{2}\right) \longrightarrow (2,2)$

The slope of BC is:

$\frac{3-1}{9-{}^{-}5} = \frac{2}{14} = \frac{1}{7}$

The perpendicular bisector must contain the point (2,2) and have slope −7.

$2 = {}^{-}7(2) + b$

$2 = -14 + b$

$16 = b$

$y = -7x + 16$

Now, set the two equations equal to each other to find the point of intersection.

$3x - 4 = {}^{-}7x + 16$

$3x = -7x + 20$

$10x = 20$

$x = 2$

If $x = 2$ then:

$y = 3(2) - 4$

$y = 6 - 4$

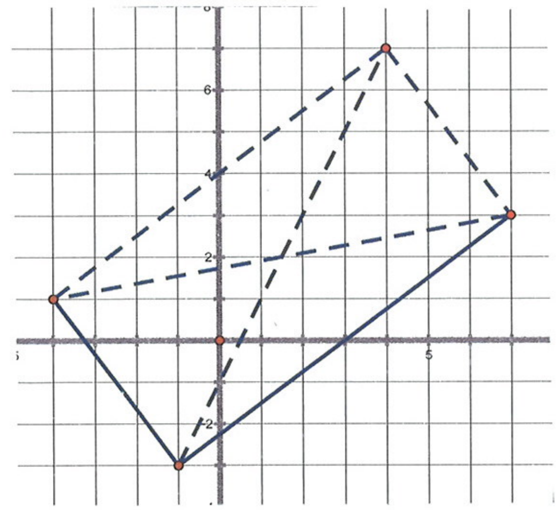
$y = 2$

(2,2)

GB

#11

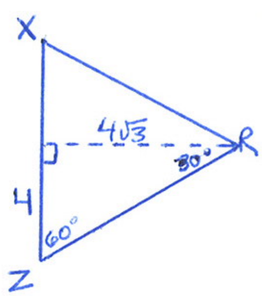
For a more visual solution, use the idea of parallel sides to find the 4th vertex of the rectangle, draw in the diagonals, and find where they cross. Or, using the fact that diagonals of a rectangle bisect each other, simply find the midpoint of the diagonal connecting (-4,1) and (7,3).



$$P = \left(\frac{-4+7}{2}, \frac{1+3}{2}\right) = (1.5, 2)$$

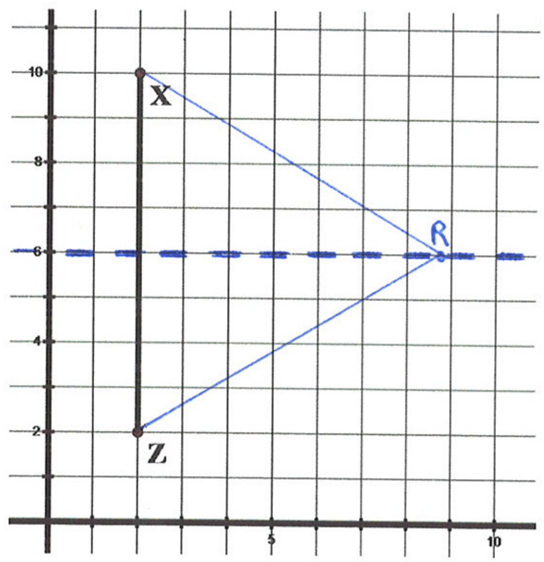
#12

If $\triangle XZR$ is equilateral, R will lie somewhere on the line $y=6$ (halfway up $\overline{XZ}$). It will also form two 30°-60°-90° triangles:



To find the length of the triangle's altitude, multiply the small leg of the 30-60-90 $\triangle$ by $\sqrt{3}$.

$4\sqrt{3} = 6.9$



Since the x-coordinate of X and Z is 2, simply add 6.9 to get the x-coordinate of R.

$R \rightarrow (8.9, 6)$

#13

Convert each population entry to scientific notation, precise to one digit (see table). Then you can identify the requested years.

1911
1921
1931
1941

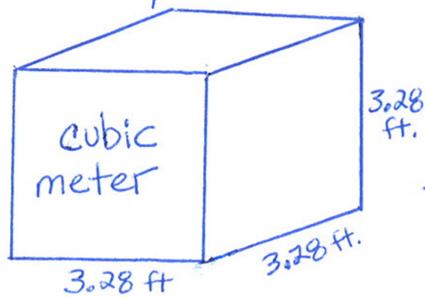
year	Population	
1901	238,396,327	2×10^8
1911	252,093,390	3×10^8
1921	251,321,213	3×10^8
1931	278,977,238	3×10^8
1941	318,660,580	3×10^8
1951	361,088,090	4×10^8
1961	439,234,771	4×10^8
1971	548,159,652	5×10^8
1981	683,329,097	7×10^8
1991	846,421,039	8×10^8
2001	1,028,737,436	1×10^9

$\left.\begin{matrix}1911\\1921\\1931\\1941\end{matrix}\right\} 3 \times 10^8$

#14 GB

In order to convert cubic meters to cubic feet, you need to know how many cubit feet are in one cubic meter.

1 meter = 3.28 feet →



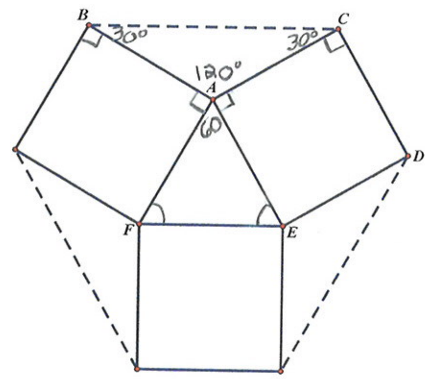
→ 1 cubic meter = 35.288 cubic feet

Volume = 35.288 ft^3

$$2.5\ m^3 \times \frac{35.288\ ft^3}{1\ m^3} = 88\ ft^3$$

#15

Notice that at each vertex of the equilateral △ (point A, for example), four angles surround the point. Two are 90° angles and one is a 60° angle. That leaves 120° for ∠BAC, which means △BAC is obtuse. Therefore, its longest side is $\overline{BC}$. Since the hexagon's sides are not all congruent (half are the length of $\overline{BC}$ and half are the length of the square's sides):



NOT EQUILATERAL

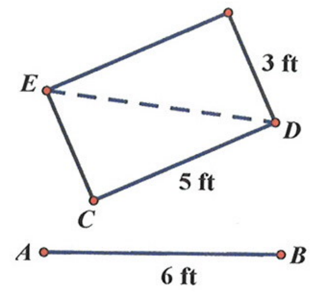
The obtuse triangles are isosceles, with base angles measuring 30°. Each of the hexagon's vertices are comprised of a 30° angle and a 90° angle. Since they are all 120°, the hexagon is:

EQUIANGULAR

#16

GB

As the box is being tipped, its tallest part will be along the diagonal. Use the Pythagorean Theorem to find out if it is less than 6 feet.



$$3^2 + 5^2 = \overline{ED}^2$$
$$9 + 25 = \overline{ED}^2$$
$$34 = \overline{ED}^2$$
$$5.83 = \overline{ED}$$

Since 5.83 is less than 6, the box can be tipped.